

Tactical Asset Allocation

Black Litterman Model with SAA as Prior

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Management Summary

A central feature of the Black-Litterman model for tactical asset allocation is the assumption that the market portfolio represents an optimal equilibrium. This is because observed market prices and capitalizations reflect an optimal balance between supply and demand, from which implied return expectations—interpreted as the average investor’s outlook—can be derived. The key innovation of the Black-Litterman framework is to combine the own subjective views about return expectation with the market equilibrium within a Bayesian setting, allowing investors to express their confidence in both sources of information. This results in posterior return estimates that form the basis of the final predictive return distribution.

A fundamental consequence of this hypothesis is the fact that assets with large market capitalization such as major equity markets (blue chips) tend to exhibit disproportionately high implied returns. Moreover, the equilibrium returns are highly sensitive to the choice of the investment universe. Changes in the set of included assets directly affect both the market portfolio weights and the covariance structure, leading to materially different implied returns. As a result, the equilibrium return vector is not invariant but depends on the modeling choices of the investor. This lack of stability raises concerns about the economic interpretability of the implied returns, as they may reflect the composition of the investment universe rather than fundamental return expectations.

At the same time, individual investors typically hold return expectations that differ from this market consensus, depending on their risk profiles and preferences. In this paper, we retain the equilibrium assumption but reinterpret it from a **strategic asset allocation (SAA) perspective** rather than from a market perspective. In other words, we replace the market-implied prior of the classical Black-Litterman model with a structurally consistent SAA-based prior that reflects both strategic return expectations and investor-specific constraints.

This shift leads to a fully **investor-centric Bayesian asset allocation framework**. The implications of this perspective for the posterior and the final return distribution are discussed in Chapter 2. Chapter 1 outlines the fundamental principles of the Black-Litterman model, while Chapter 3 presents an empirical case study comparing the traditional and the SAA-based approaches.

1 Black-Litterman Model: a brief description

1.1 The „Prior“ Component

Let $U(x)$ be the utility function of an investor with portfolio weights x , λ his risk aversion and the vector of his expected returns r . Σ represents the empirical covariance matrix. $U(x) = r'x - \lambda x' \Sigma x$. Assuming that the market portfolio w_m is optimal, it must satisfy the first-order condition: $U'(x) = r - 2\lambda \Sigma x = 0$. This leads to the implied equilibrium returns: $\Pi = 2\lambda \Sigma w_m$. The prior distribution is then specified as: $\mu \sim N(\Pi, C = \tau \Sigma)$. The matrix C is the covariance matrix of the prior and describes the variance of the unknown vector μ around the prior mean Π . C describes the degree to which the investor considers the prior Π to be uncertain. The scaling by τ ensures that the market covariance Σ is transferred to the prior uncertainty. Typically, τ is small. The parameter λ represents the risk aversion of the average investor and is derived from the capital asset pricing model $^2 \lambda = \frac{\text{market_excess_return}}{2 * \text{market_variance}}$.

1.2 The „Views“ Component

The investor typically has his own return expectations and deviates from the prior Π according to the matrix equation: $q = P\mu + \varepsilon$, where $\varepsilon \sim N(0, \Omega)$. The matrix Ω measures how precise or imprecise the investor's own views are. Small diagonal elements in Ω indicate that the corresponding view is held with a high degree of certainty. Large diagonal elements mean that the view is uncertain and should be assigned only a small weight in the posterior. If Ω is diagonal, the views are modeled as mutually uncorrelated. A non-diagonal Ω would additionally allow the errors of multiple views to be correlated with one another.

1.3 The „Posterior“ Component

The combination of two different sources of opinion regarding expected returns can be calculated using conditional probabilities as the product of the prior and the views:

The Bayes-Formula connects these two distributions: $p(\mu | q) \propto p(q | \mu) * p(\mu)$

where $p(q | \mu) \propto \exp\{-1/2 (q - P.\mu)' \Omega^{-1} (q - P.\mu)\}$ and $p(\mu) \propto \exp\{-1/2 (\mu - \Pi)' C^{-1} (\mu - \Pi)\}$.

If we add the quadratic terms in the exponent, we get the following form:

$$-1/2 [\mu'(P'\Omega^{-1}P + C^{-1})\mu - 2\mu'(P'\Omega^{-1}q + C^{-1}\Pi)]$$

Combining Prior and Views we get the Posterior as a new normal distribution.

That's the actual Black-Litterman update: $(\mu | q) \sim N(\mu_{BL}, \Sigma_{BL})$

where: $\Sigma_{BL} = (C^{-1} + P'\Omega^{-1}P)^{-1}$ and $\mu_{BL} = (C^{-1} + P'\Omega^{-1}P)^{-1} (C^{-1}\Pi + P'\Omega^{-1}q)$

It is immediately apparent that C and Ω appear symmetrically in the Bayesian update, but they originate from different model components. The posterior Mean μ_{BL} is a weighted compromise between the market prior Π and the views q . The weights are derived from the precision matrices or inverses of the uncertainty matrices C^{-1} and Ω^{-1} . The limiting cases make the roles of C and Ω particularly transparent.

Case A: $\Omega \rightarrow 0$ Then the views are treated as exact.

Case B: $\Omega \rightarrow \infty$ Then the views are effectively ignored and μ_{BL} approaches Π .

Case C: $\tau \rightarrow 0$ Then $C = \tau \Sigma \rightarrow 0$ the prior becomes precise and Π dominates.

Case D: τ is high Then the prior becomes uncertain and the views gain more influence

¹ In Meucci-Paper $\tau = 1/(\text{number of observation})$ *The Black-Litterman Approach: Original Model and Extension (Attilio Meucci, October 12, 2010)*

² This comes from maximizing the mean-variance utility function, where the slope of the efficient frontier (market price of risk) reflects how much return investors require per unit of risk—i.e., their average risk aversion

1.4 The finale Forecast-Distribution

After the Bayesian update, μ itself is still a random vector with posterior covariance Σ_{BL} .

However, for portfolio optimization, what matters are the distributions of future returns R.

$$R \mid \mu \sim N(\mu, \Sigma)$$

$$\mu \mid q \sim N(\mu_{BL}, \Sigma_{BL})$$

Using the law of total variance, we obtain:

$$E[R] = \mu_{BL}$$

$$\text{Var}(R) = E[\text{Var}(R \mid \mu)] + \text{Var}(E[R \mid \mu]) = \Sigma + \Sigma_{BL}$$

Σ describes the variation of realized returns around the true mean.

Σ_{BL} describes the uncertainty regarding where this Mean actually lies.

1.5 Illustrative examples

A. Views with high confidence

Input					Output									
	Strat. Return	Volatility	Views	Uncertainty	Covariance matrix (Σ)	Asset 1	Asset 2	Asset 3	Inverse matrix (Σ^{-1})	Asset 1	Asset 2	Asset 3		
Asset 1	3,0%	5,0%	2,0%	0,001%	Asset 1	0,00250	0,00018	0,00060	Asset 1	408	61	-49		
Asset 2	5,5%	7,0%	-2,0%	0,001%	Asset 2	0,00018	0,00490	0,00756	Asset 2	61	1.083	-571		
Asset 3	9,0%	12,0%	-3,0%	0,001%	Asset 3	0,00060	0,00756	0,01440	Asset 3	-49	-571	371		
Correlation matrix					Uncertainty matrix (Ω)					Inverse matrix (Ω^{-1})				
	Asset 1	Asset 2	Asset 3		EUR Treasury	0,001%	0,000%	0,000%	Asset 1	100.000	0	0		
Asset 1	1	0,05	0,10		US Treasury	0,000%	0,001%	0,000%	Asset 2	0	100.000	0		
Asset 2	0,05	1	0,90		EUR Corporate	0,000%	0,000%	0,001%	Asset 3	0	0	100.000		
Asset 3	0,10	0,90	1		Marktparameter					Returns				
Market Allocation					Marktrendite					Views	Prior	BL		
Asset 1	50,0%			Tau	R _m	4,95%			Asset 1	2,00%	2,59%	2,07%		
Asset 2	30,0%		0,50%		Markt-Volatilität	σ_m	5,22%		Asset 2	-2,00%	5,58%	-1,04%		
Asset 3	20,0%				Risikoaversion	λ	9,09		Asset 3	-3,00%	9,91%	-1,87%		

Result: The Black-Litterman vector is more heavily weighted toward its own views

B. Views with high uncertainty

Input					Output									
	Strat. Return	Volatility	Views	Uncertainty	Covariance matrix (Σ)	Asset 1	Asset 2	Asset 3	Inverse matrix (Σ^{-1})	Asset 1	Asset 2	Asset 3		
Asset 1	3,0%	5,0%	2,0%	99,000%	Asset 1	0,00250	0,00018	0,00060	Asset 1	408	61	-49		
Asset 2	5,5%	7,0%	-2,0%	99,000%	Asset 2	0,00018	0,00490	0,00756	Asset 2	61	1.083	-571		
Asset 3	9,0%	12,0%	-3,0%	99,000%	Asset 3	0,00060	0,00756	0,01440	Asset 3	-49	-571	371		
Correlation matrix					Uncertainty matrix (Ω)					Inverse matrix (Ω^{-1})				
	Asset 1	Asset 2	Asset 3		EUR Treasury	99,000%	0,000%	0,000%	Asset 1	1	0	0		
Asset 1	1	0,05	0,10		US Treasury	0,000%	99,000%	0,000%	Asset 2	0	1	0		
Asset 2	0,05	1	0,90		EUR Corporate	0,000%	0,000%	99,000%	Asset 3	0	0	1		
Asset 3	0,10	0,90	1		Marktparameter					Returns				
Market Allocation					Marktrendite					Views	Prior	BL		
Asset 1	50,0%			Tau	R _m	4,95%			Asset 1	2,00%	2,59%	2,59%		
Asset 2	30,0%		0,50%		Markt-Volatilität	σ_m	5,22%		Asset 2	-2,00%	5,58%	5,58%		
Asset 3	20,0%				Risikoaversion	λ	9,09		Asset 3	-3,00%	9,91%	9,91%		

Result: The Black-Litterman vector is based exclusively on prior returns.

2 Black-Litterman Model with SAA-Prior

2.1 The mathematical Framework

We consider a market with n Assets and a λ -Investor holding a portfolio x who is aiming to maximize his Utility function U under budget constraint ⁴ $1^T x = 1$ and some equality investment ⁵ $Ax = b$ restrictions.

This investment problem can be formulated mathematically as follows:

$$U(x) = r'_{SAA} x - \lambda x' \Sigma_{SAA} x$$

$x \in \mathbb{R}^n$: The vector of portfolio-weights

$r \in \mathbb{R}^n$: The vector of long-term (SAA) expected returns

$\lambda > 0$: The investor's risk aversion

$\Sigma_{SAA} \in \mathbb{R}^{n \times n}$: The empirical covariance matrix

$A \in \mathbb{R}^{n \times m}$: The Matrix of investment constraints

$b \in \mathbb{R}^m$: The target vector of the constraints.

The optimal portfolio x^* is given by:

$$x^* = \frac{1}{2\lambda} \Sigma_{SAA}^{-1} (r_{SAA} + \gamma \mathbf{1} + A\eta)$$

The multipliers γ and η are determined by the Lagrange system of linear equations:

$$\begin{pmatrix} 1^T \Sigma_{SAA}^{-1} \mathbf{1} & 1^T \Sigma_{SAA}^{-1} A \\ A^T \Sigma_{SAA}^{-1} \mathbf{1} & A^T \Sigma_{SAA}^{-1} A \end{pmatrix} \begin{pmatrix} \gamma \\ \eta \end{pmatrix} = \begin{pmatrix} 2\lambda - 1^T \Sigma_{SAA}^{-1} r_{SAA} \\ 2\lambda b - A^T \Sigma_{SAA}^{-1} r_{SAA} \end{pmatrix}$$

Proof:

To solve this problem, we use the ⁶Lagrange-Function:

$$L(x, \gamma, \eta) = r_{SAA}^T x - \lambda x^T \Sigma_{SAA} x + \gamma (1^T x - 1) + \eta^T (A^T x - b)$$

The first-order optimality condition follows from the gradient with respect to x :

$$\nabla_x L = r_{SAA} - 2\lambda \Sigma_{SAA} x + \gamma \mathbf{1} + A\eta = 0$$

It follows that: $2\lambda \Sigma_{SAA} x = r_{SAA} + \gamma \mathbf{1} + A\eta$. And with that: $x^* = \frac{1}{2\lambda} \Sigma_{SAA}^{-1} (r_{SAA} + \gamma \mathbf{1} + A\eta)$

Substituting the expression for x into the constraints (budget constraint $1^T x = 1$ and investment constraint: $A^T x = b$) gives the two equations for the Lagrange multipliers γ and η .

$$\frac{1}{2\lambda} \mathbf{1}^T \Sigma_{SAA}^{-1} (r_{SAA} + \gamma \mathbf{1} + A\eta) = 1$$

$$\frac{1}{2\lambda} A^T \Sigma_{SAA}^{-1} (r_{SAA} + \gamma \mathbf{1} + A\eta) = b$$

A smart algebraic rearrangement of these two equations yields exactly the Lagrange system of linear equations.

³ The optimization framework follows standard quadratic programming formulations as discussed in Boyd and Vandenberghe (2004)

⁴ Budget-Restriction

⁵ Here we consider only the equation constraints, since the solution has a closed-form expression. For optimization problems with inequality constraints ($Cx < d$), the solution has a Kuhn-Tucker representation. This means that the Lagrange multipliers are also optimized alongside x .

⁶ The Lagrange function transforms a constrained optimization problem into an unconstrained one, using a penalty coefficient for each constraint

2.2 The implied Risk Aversion

The optimization routine described in Section 2.1 is performed for various levels of risk aversion (λ). This results in a (μ -sigma) efficiency frontier that corresponds to all optimal lambda portfolios. The investor is invited to select a suitable target portfolio. Various selection criteria can be used for this purpose, such as return target, risk limit, Roy, or Kataoka. In our example, we will take the lambda portfolio with the highest Sharpe ratio as the target portfolio. After selecting a target portfolio x^* , we get: ${}^7U^* = r'_{SAA}x^* - \lambda^*x^{*\prime}\Sigma_{SAA}x^*$ and its corresponding Risk Aversion: $\lambda^* = \frac{r'_{SAA}x^* - U^*}{x^{*\prime}\Sigma_{SAA}x^*}$

2.3 The „SAA-Prior“ Component

The implied strategic returns are used as the prior mean in a Bayesian model. This makes the investor's own SAA the starting point for the Black-Litterman update. The SAA prior is chosen as $\mu \sim N(r_{SAA}, C_{SAA})$. The Matrix $C_{SAA} = \tau\Sigma_{SAA}$ is the corresponding covariance matrix and describes solely the extent to which the investor views the SAA prior r_{SAA} as uncertain. The scaling via τ ensures that the market covariance Σ_{SAA} is transferred to the prior uncertainty. Typically, τ is small, for example $\tau = 1/T$.

2.4 The „Views“ Component

The investor typically has their own return expectations and deviates from the prior Π according to the matrix equation: $q = P\mu + \varepsilon$, where $\varepsilon \sim N(0, \Omega)$. The matrix Ω measures how precise or imprecise the investor's own views are. Small diagonal elements in Ω indicate that the corresponding view is held with a high degree of certainty. Large diagonal elements mean that the view is uncertain and should be assigned only a small weight in the posterior. If Ω is diagonal, the views are modeled as mutually uncorrelated. A non-diagonal Ω would additionally allow the errors of multiple views to be correlated with one another.

2.5 The „Posterior“ Component

Using Prior and Views, the posterior returns are derived via Bayes' theorem. These combine the strategic SAA perspective with the tactical views in a single, consistent step.

$$\begin{aligned}\mu \mid q &\propto N(\mu_{BL}^{SAA}, \Sigma_{BL}^{SAA}) \\ \Sigma_{BL}^{SAA} &= (C^{-1} + P^T \Omega^{-1} P)^{-1} \\ \mu_{BL}^{SAA} &= (C^{-1} + P^T \Omega^{-1} P)^{-1} (C^{-1} r_{SAA} + P^T \Omega^{-1} q)\end{aligned}$$

2.6 The Final Forecast Distribution

For portfolio optimization, it is not only the posterior distribution of μ that is relevant, but also the predictive distribution of future returns R . This distribution incorporates both market risk and uncertainty regarding expected returns $E[R] = \mu_{BL}^{SAA}$ und ${}^8\text{Var}(R) = \Sigma_{SAA} + \Sigma_{BL}^{SAA}$

⁷ Since the optimal solution satisfies the constraints, the terms with Lagrange multipliers in U^* are equal to zero

⁸ Law of total variance

2.7 Analogy with traditional Black-Litterman Model

As we just saw, the central idea of this approach is to formulate the equilibrium assumption not at the market level, but at the investor level. Specifically, it is assumed that the investor faces a constrained optimization problem of the form $\{r'_{SAA}x - \lambda x' \Sigma_{SAA} x\} \rightarrow \text{Max}$, under $1'x = 1$ & $Ax=b$. The optimal portfolio x^* is thus no longer determined by market capitalization, but rather by the investor's strategic return assumptions and constraints. The first-order optimality condition yields: $r_{SAA} - 2\lambda \Sigma_{SAA} x^* + \gamma 1 + A\eta = 0$. which yields the implied return vector: $r_{SAA} = 2\lambda \Sigma_{SAA} x^* - \gamma 1 - A\eta$. While in the standard Black-Litterman model implied returns are derived from an unrestricted market equilibrium, here they arise from a restricted investor equilibrium that explicitly considers investment constraints and objectives. This has the following implications:

1. The prior is no longer market-based but investor-specific, reflecting strategic expectations and constraints.
2. Risk aversion is determined endogenously from the optimization problem rather than calibrated exogenously from market data.
3. Strategic allocation and tactical adjustments are consistently integrated within a unified Bayesian framework.

The following table compares the model features between the classic Black-Litterman approach and its variant using the SAA as a prior:

	Black-Litterman	Black-Litterman with SAA-Prior
Optimization problem:	$r'x - \lambda x' \Sigma x \rightarrow \text{Max}$	$r'_{SAA}x - \lambda x' \Sigma_{SAA} x \rightarrow \text{Max}$, such that: $1'x = 1$ & $Ax=b$
Utility & Lagrange Function	$U = r'x - \lambda x' \Sigma x$	$L = r'_{SAA}x - \lambda x' \Sigma_{SAA} x + \gamma(1'x - 1) + \eta^T(A^T x - b)$
Optimality condition: $L'(x)=0$	$U'(x) = 0 \Rightarrow r - 2\lambda \Sigma x = 0$	$L'(x) = 0 \Rightarrow r_{SAA} - 2\lambda \Sigma_{SAA} x + \gamma 1 + A\eta = 0$
Reference portfolio	Market portfolio: $x^* = w_m$	$x^* = \frac{1}{2\lambda} \Sigma_{SAA}^{-1} (r_{SAA} + \gamma 1 + A\eta)$
Implied return expectation	$\Pi = 2\lambda \Sigma w_m$	$r_{SAA} = 2\lambda \Sigma_{SAA} x^* - \gamma 1 - A\eta$
Risiko-Aversion: λ	$\frac{r'_{Market} w_m - r_f}{2 * w_m' \Sigma w_m}$	$\frac{r'_{SAA} x^* - L^*}{x^{*T} \Sigma_{SAA} x^*}$
Prior: μ	$N(\Pi, C = \tau \Sigma)$	$N(r_{SAA}, C_{SAA} = \tau \Sigma_{SAA})$
Views: q	$P\mu + \epsilon$, where $\epsilon \sim N(0, \Omega)$	$P\mu + \epsilon$, wobei $\epsilon \sim N(0, \Omega)$
Posterior: (μq)	$N(\mu_{BL}, \Sigma_{BL})$ $\mu_{BL} = (C^{-1} + P' \Omega^{-1} P)^{-1} (C^{-1} \Pi + P' \Omega^{-1} q)$ $\Sigma_{BL} = (C^{-1} + P' \Omega^{-1} P)^{-1}$	$N(\mu_{BL}^{SAA}, \Sigma_{BL}^{SAA})$ $\mu_{BL}^{SAA} = (C^{-1} + P^T \Omega^{-1} P)^{-1} (C^{-1} r_{SAA} + P^T \Omega^{-1} q)$ $\Sigma_{BL}^{SAA} = (C^{-1} + P^T \Omega^{-1} P)^{-1}$
Final Forecast Distribution	$N(\mu_{BL}, \Sigma + \Sigma_{BL})$	$N(\mu_{BL}^{SAA}, \Sigma_{SAA} + \Sigma_{BL}^{SAA})$

3 Case Study

3.1 Input data

This section illustrates the practical implications of the proposed SAA-based Black-Litterman model compared to the classic Black-Litterman approach. The analysis is based on a multi-asset portfolio consisting of government bonds, corporate bonds, and stocks from us and euro area. The covariance matrix, the SAA-⁹Returns, the corresponding Volatilities and correlations were estimated from historical ¹⁰data. In addition, portfolio constraints such as minimum and maximum weights and linear constraints are considered. In this study, the views are entered individually and with a high degree of certainty. The Views could be linked to an economic ¹¹Risk-On Scenario in which government bonds tend to underperform while corporate bond and stock prices rise.

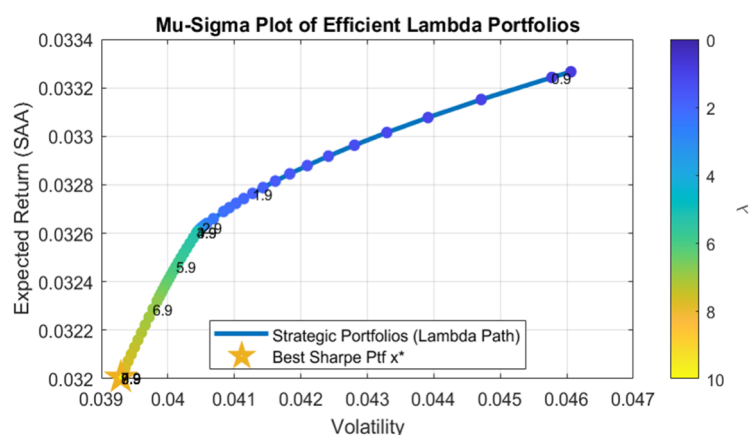
	Benchmark	Market Ptf	SAA-Return	Volatility	Min	Max
EUR_Treasury	LEATTREU Index	20%	3,1%	4,5%	35%	60%
US_Treasury	LUATTREH Index	30%	2,5%	4,7%	10%	20%
EUR_Corporate	LECP TREU Index	7%	3,4%	4,1%	20%	40%
US_Corporate	LUACTREH Index	16%	3,7%	6,1%	5%	10%
EUR_Equity	SXXE Index	26%	4,0%	17,2%	5%	15%

P Matrix	EUR_Treasury	US_Treasury	EUR_Corporate	US_Corporate	EUR_Equity	Views	Uncertainty
EUR_Treasury	1	0	0	0	0	-1,00%	0,01%
US_Treasury	0	1	0	0	0	-2,00%	0,01%
EUR_Corporate	0	0	1	0	0	4,00%	0,01%
US_Corporate	0	0	0	1	0	4,50%	0,01%
EUR_Equity	0	0	0	0	1	7,00%	0,01%

The market portfolio used in this study is based on the ¹²current market capitalization.

3.2 Lambda Efficient Frontier

The constrained optimization problem is solved using a grid of risk aversion levels λ , resulting in a μ - σ efficiency frontier. Each point corresponds to an optimal portfolio for a specific risk level. A target portfolio x^* is selected from this efficient set, in this case based on the maximum Sharpe ratio. The corresponding λ^* represents the investor's specific risk aversion.



⁹ Empirical average returns

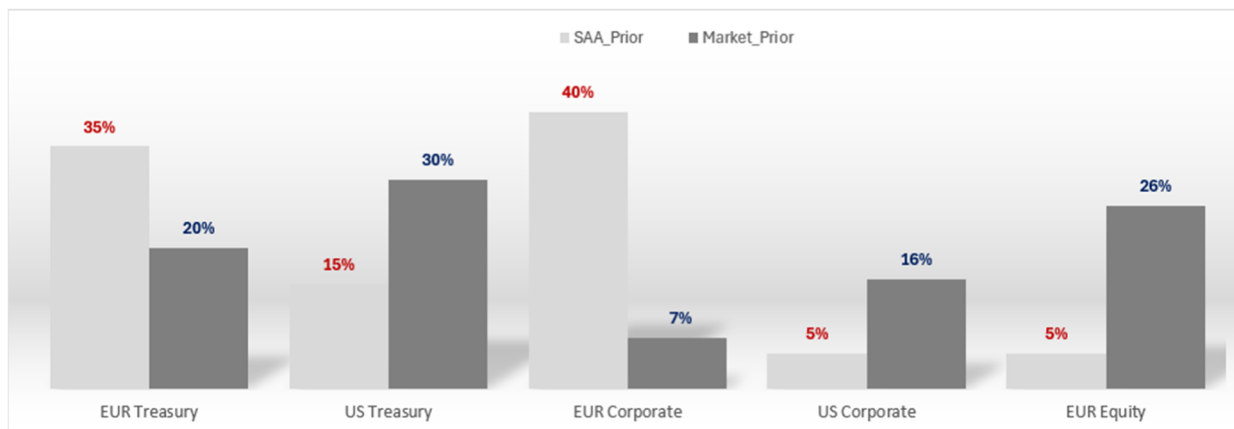
¹⁰ The empirical data consist of monthly time series of total return indices dating back to December 1998

¹¹ For example, an economic Expansion

¹² Data as of March 31, 2026

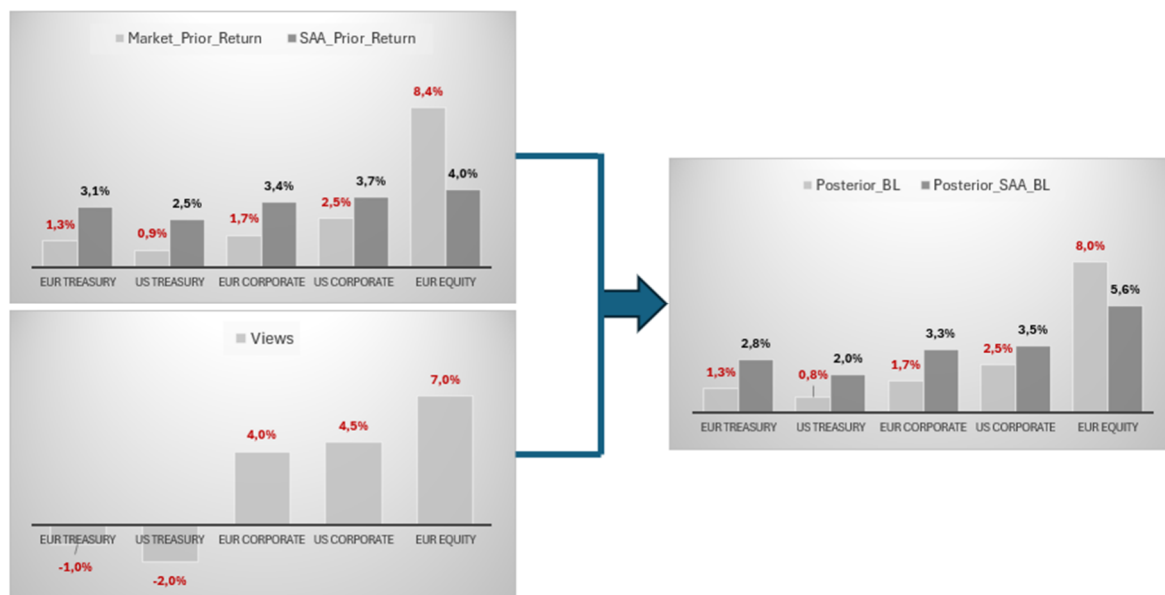
3.3 Prior portfolios

As expected, the SAA benchmark differs significantly from the market portfolio.



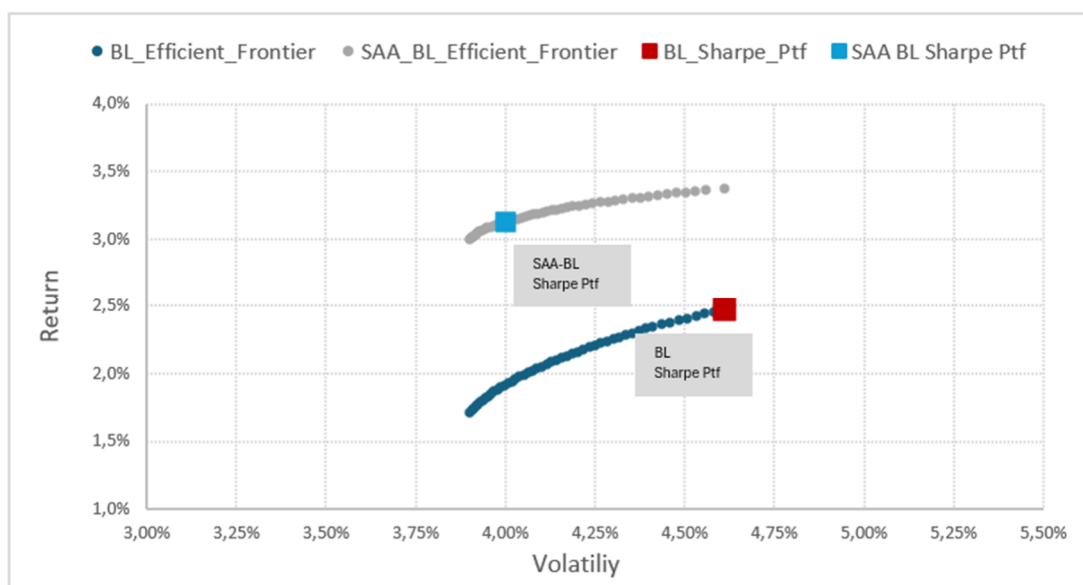
3.4 Prior, Views and Posterior returns

The implied equilibrium returns are biased toward the structure of the Market portfolio, which may not be appropriate for an investor with diversification objectives or specific constraints.

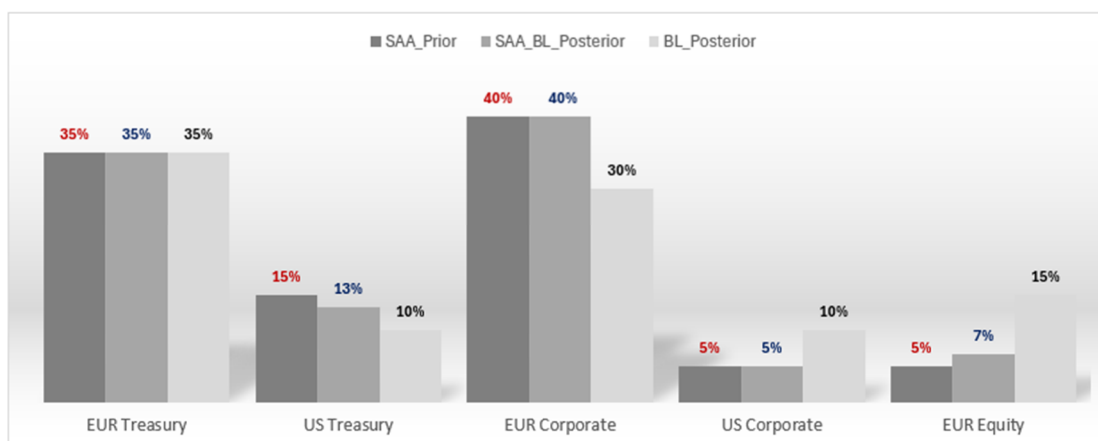


The results highlight the dominant role of the views when their uncertainty is very low. In this setting, the posterior returns are largely driven by the views, with the prior mainly determining the baseline level. In the classical Black-Litterman model, the already elevated market-implied prior for equities leads to persistently high posterior returns, even under strong view dominance. This reflects the structural bias introduced by market-capitalization weighting. In contrast, the SAA-based approach results in a more balanced adjustment. Starting from a moderate strategic prior, the model incorporates the views in a controlled manner, leading to a significant but economically plausible increase in expected returns. This demonstrates that even under extreme confidence in the views, the choice of prior remains a key determinant of the final outcome.

3.5 Black-Litterman portfolio optimization



The SAA-BL efficient frontier lies above the traditional BL efficient frontier. The incorporation of strategic return assumptions and constraints results in structurally more efficient portfolios. In particular, the SAA model's maximum Sharpe ratio portfolio delivers a higher return while carrying lower risk.



The portfolio comparison highlights a key advantage of the SAA-based framework. While the classical Black-Litterman model leads to substantial reallocations, most notably a significant shift from corporate bonds to equities, the SAA-based approach preserves the overall strategic structure.

In particular, the classical model increases equity exposure from 5% to 15% and reduces core bond allocations, reflecting the strong influence of market-implied returns. In contrast, the SAA-based portfolio remains closely aligned with the strategic allocation, with only moderate and targeted adjustments.

This demonstrates that the proposed framework enables the integration of tactical views without disrupting the underlying strategic allocation, thereby ensuring stability and pragmatic portfolio steering.

4 Conclusion

An important aspect of the classical Black-Litterman framework is its reliance on the market portfolio as the reference point. Since the implied equilibrium returns are derived from market-capitalization-weighted portfolios, large and dominant asset classes—such as blue-chip equities—tend to exhibit disproportionately high expected returns. In addition, the resulting equilibrium returns are highly sensitive to the definition of the investment universe. Changes in the set of included assets directly affect both the portfolio weights and the covariance structure, leading to unstable and non-invariant return estimates. This dependence on market structure rather than fundamental expectations can distort the economic interpretation of the model's outputs.

This paper examines the Black-Litterman framework from a perspective focused on the investor's profile and objectives rather than on a global and heterogeneous market by introducing an investor-specific equilibrium based on strategic asset allocation (SAA). While the traditional approach relies on market-implied returns, the proposed model anchors the entire Bayesian updating process in the investor's own strategic framework.

The key advantage of this approach is not only to get a realistic and economic risk-return efficiency, but, more importantly, a **high degree of consistency between strategic and tactical asset allocation**.

This is particularly relevant for institutional investors and benchmark-driven mandates, as the proposed framework ensures that tactical adjustments are implemented in alignment with the strategic allocation, while maintaining disciplined control over tracking error and portfolio turnover.

In the proposed framework, tactical portfolio adjustments are performed **in full alignment with the strategic allocation**, rather than deviating from it. This ensures that portfolio decisions remain consistent with long-term investment objectives, while still allowing for the incorporation of short-term views.

From a practical perspective, this leads to several important benefits:

- Tactical allocation is conducted **in tandem with the strategic allocation**, rather than global market structure
- Portfolio adjustments naturally respect **tracking error constraints and turnover limits**
- Strategic return assumptions can be updated dynamically to reflect changing economic conditions, without breaking the consistency of the overall framework
- The model provides a unified structure that integrates **strategy, views, and constraints** into a single coherent process

In contrast to the traditional Black-Litterman model, which may lead to allocations that drift away from the investor's strategic positioning, the proposed SAA-based approach ensures that all portfolio decisions remain anchored in the investor's long-term framework.

Overall, the model should be interpreted not merely as an optimization tool, but as a **consistent decision-making framework** that bridges the gap between strategic planning and tactical portfolio implementation and ultimately, the model reconsiders Black-Litterman as an investor-centric framework in which strategy and tactical views are no longer competing forces, but fully integrated components of a consistent allocation process.

5 Main References

- Boyd, S., & Vandenberghe, L. (2004). *Convex Optimization*. Cambridge University Press
- Black & Litterman (1992) Global Portfolio Optimization
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